PROJECTED WRITTEN NOTES FROM THE VIDEO

# CONIC SECTIONS: HYPERBOLAS

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**Hyperbolas:** $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ OR $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

$a^2$ is the Denominator under the added squared variable.

The Foci and the Vertices are on the axis of the added squared variable.

$c^2 = a^2 + b^2$ is the case for Hyperbolas

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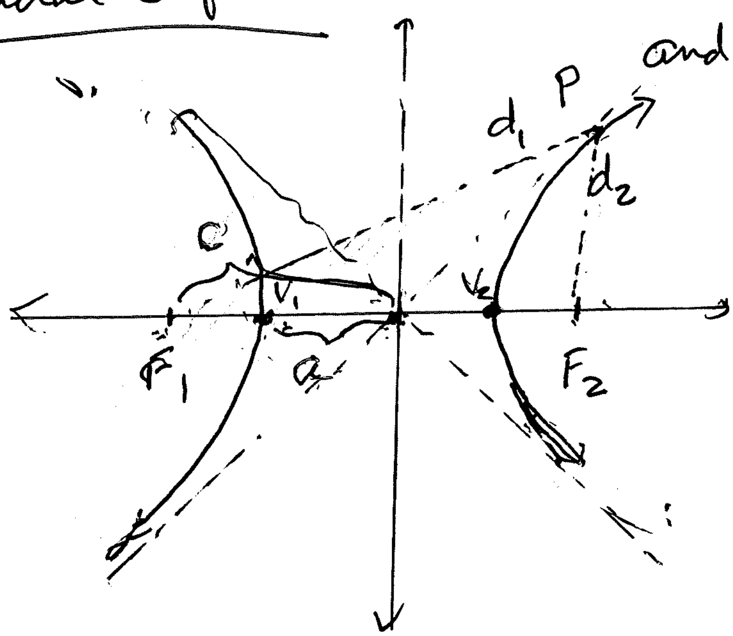
**General Definition** Given Two points $F_1, F_2$ (Foci) and a number $L = 2a$, Point P has $|d_1 - d_2| = L = 2a$



# Hyperbolas

**7** The hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

has foci $(\pm c, 0)$, where $c^2 = a^2 + b^2$, vertices $(\pm a, 0)$, and asymptotes $y = \pm(b/a)x$.

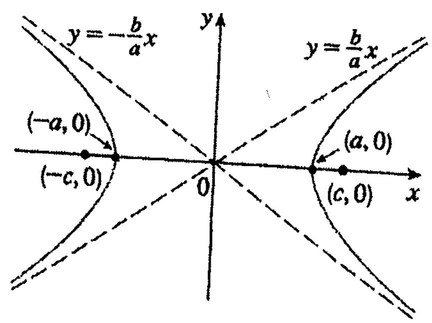



**FIGURE 12**
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

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**8** The hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

has foci $(0, \pm c)$, where $c^2 = a^2 + b^2$, vertices $(0, \pm a)$, and asymptotes $y = \pm(a/b)x$.

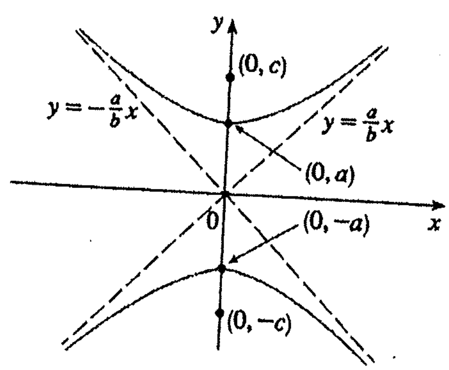


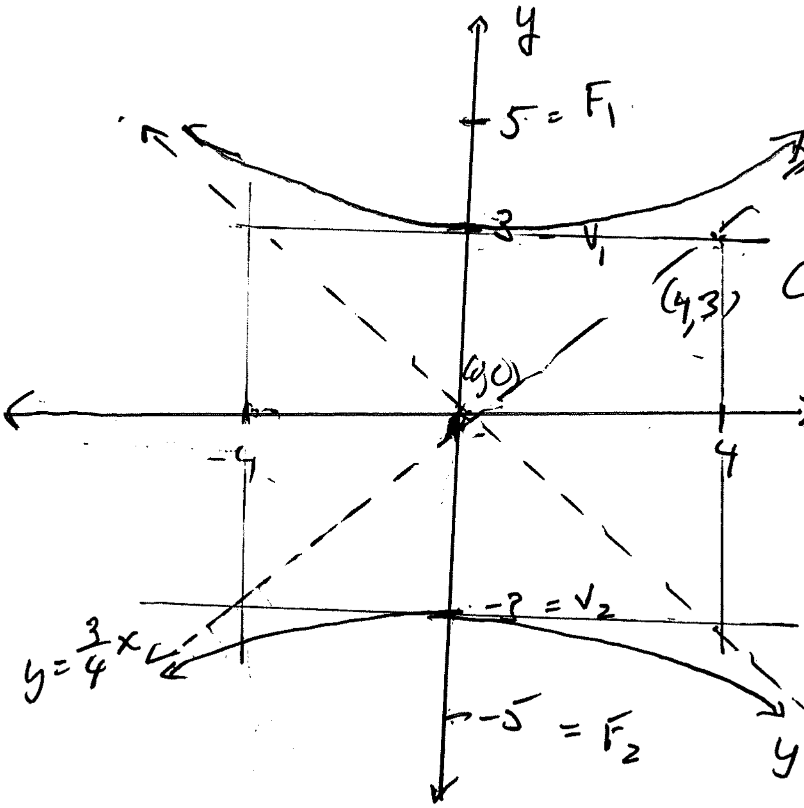

**FIGURE 13**
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

**Problem:** Find the Foci and Vertices and sketch the curve of

$$16y^2 - 9x^2 = 144$$

**Sol'n** Divide by $144 = 16 \times 9$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$




$a^2 = 9 \Rightarrow a = 3$

$b^2 = 16 \Rightarrow b = 4$

$c^2 = a^2 + b^2 = 9 + 16 = 25$

$c = 5$

The Foci are

$(0,5)$ and $(0,-5)$

The Vertices are

$(0,3)$ and $(3,0)$

Slope $= \frac{3}{4}$

Equations of Asymptotes

$y = \frac{3}{4}x$ and

# DRAWING THE BOX

## Hyperbolas

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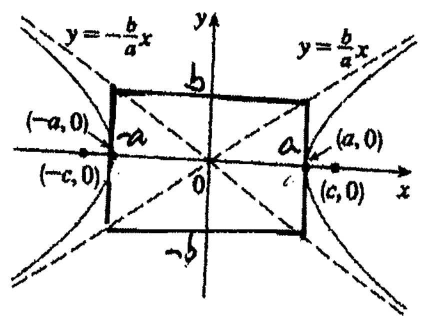


**FIGURE 12**
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

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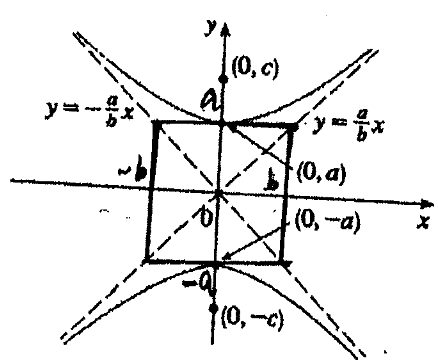



**FIGURE 13**
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

# Summary of the standard Equation Forms of Conics (UNSHIFTED)

## PARABOLAS:

$x^2 = 4py$, $p \neq 0$ OR $y^2 = 4px$, $p \neq 0$

| | |
|---|---|
| THE FOCUS is $(0, p)$. | THE FOCUS is $(p, 0)$. |
| THE Directrix is "$y = -p$." | THE Directrix is "$x = -p$." |

In BOTH CASES: THE VERTEX is the Origin $(0,0)$.

THE FOCUS is on the axis of the Degree 1 variable.

$|p|$ = The Distance: VERTEX TO FOCUS.

## FOR ELLIPSES AND HYPERBOLAS (BOTH)

$a$ = the DISTANCE: CENTER TO EACH VERTEX.

$c$ = the DISTANCE: CENTER TO EACH FOCUS.

THE FOCI AND VERTICES ARE ON THE AXIS OF the Squared Variable that is over $a^2$.

## ELLIPSES:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{OR} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{where} \begin{cases} a > b > 0 \\ c^2 = a^2 - b^2 \\ c < a \end{cases}$$

$a^2$ is the LARGER DENOMINATOR.

The FOCI AND VERTICES LIE ON THE AXIS of the Variable OVER $a^2$.

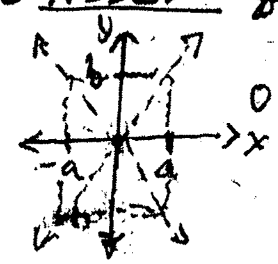
## HYPERBOLAS:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{OR} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{where} \begin{cases} c^2 = a^2 + b^2 \\ c > a \end{cases}$$

$a^2$ is the denominator under the ADDED Squared Variable.

THE FOCI ARE ON THE AXIS OF THE ADDED Squared Variable.

The asymptotes are lines along the DIAGONALS of the "BOX"




OR